

Answer the following questions. Each question counts for 10 marks. Calculators are not allowed.

1. a) Solve $x^3 - 2x^2 > 2 - x$.

b) Find $\lim_{x \rightarrow 1^-} \frac{[x - 1]}{|x^2 - 1|}$.

2. a) Prove that the product of two odd functions is an even function.

b) Let

$$g(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

and $h(x) = -x^2 + 3x - 7$. Sketch the graph of $f(x) = (g \circ h)(x)$.

3. a) Let $x = \tan \frac{\theta}{2}$. Find the function $f(x)$ such that $\sin \theta = f(x)$.

b) Find the equations of the straight lines that pass through the point $(5,0)$ and are tangent to the circle $x^2 + y^2 = 9$.

4. a) Let $f(x) = \frac{\sqrt[3]{x^6 + 1}}{4x^2 - 1}$. Find the vertical and horizontal asymptotes of f .

b) Study the continuity of the function

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational.} \end{cases}$$